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Convergence acceleration of a nonlinear solver using statistical learning

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Introduction

- Numerical simulations in industry : crucial to help decision making.
- Traditional iterative algorithms : convergence heavily dependent on initialization.
- This work explores alternative methods to improve initialization, accelerating convergence while preserving physical accuracy.

Problem definition

Nonlinear PDE → high-dimensional nonlinear algebraic equations, solved using a Quasi-Newton method.

Finding $U^\star$ such that $F(U^\star, \lambda) = 0$
where λ is a vector of parameters.

Applications

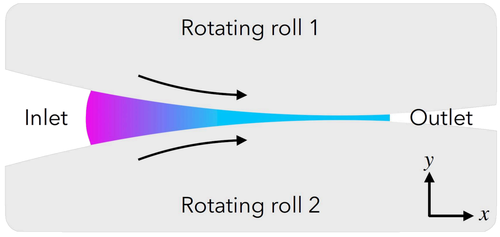
• 1D Nonlinear Poisson equation

Let Ω be an open domain of $\mathbb{R}$.
The nonlinear Poisson equation is formalised as follows:

$$\begin{cases} -\frac{\partial}{\partial x}\left[q(u)\frac{\partial}{\partial x}u\right] = g(x,\lambda) & \text{in } \Omega \\ u = u_D & \text{on } \partial\Omega \end{cases}.$$

• 2D calendering process

Rubber process manufacturing : compress material between two counter-rotating rolls.
Focus on mechanical fields. Heat transfers are not considered in this study.



Governing equations:
Mass and momentum conservation for incompressible highly viscous flow : divergence free velocity and nonlinear Stokes equations.
Nonlinear power-law viscosity model : λ contains the coefficients of the viscosity model.

Strategy

Use machine learning to find a new initialization for the nonlinear solver generating less iterations to converge.

Data & baselines

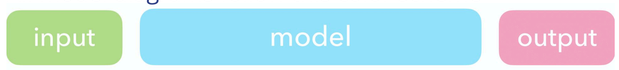
Dataset:
a set of vectors $\{\lambda_k\}$ of parameters
a set of associated discretized solutions $\{U_k^\star\}$ computed using the solver.

Sampling strategy:
Train: regular grid 10 × 10 in the parameter space.
Test: 100 pairs via Latin Hypercube Sampling in the same intervals.

2 baseline strategies:
Classical initialization: constant field set to boundary value.
Nearest initialization: use the solution from the closest parameter in the database.

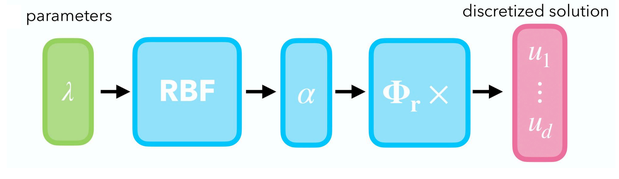
Tested methods

All model diagrams follow this color scheme

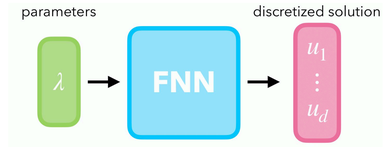


• Proper Orthogonal Decomposition (POD) [4]

Consider a snapshot matrix $U = \begin{bmatrix} | & | & & | \\ U_1^\star & U_2^\star & \cdots & U_n^\star \\ | & | & & | \end{bmatrix}$.
Compute truncated SVD ($U = \Phi_r \Sigma_r V_r^T$) to obtain Φ_r , the orthonormal reduced basis between projection coefficients α_k and solution U_k .
Define interpolation method like Radial Basis Function (RBF) between parameters and projection coefficients.



• Fully connected Neural Networks (FNN)



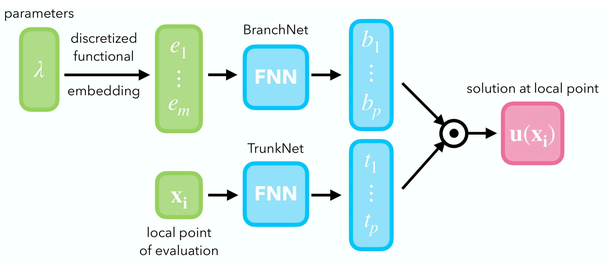
Training objective (MSE):

$$\min_\theta \ \mathcal{L}(\theta) = \frac{1}{n}\sum_{k=1}^{n} \|\mathrm{FNN}_\theta(\lambda_k) - U_k\|^2$$

This architecture is poorly scalable one output per discretization point, and prediction points are fixed.

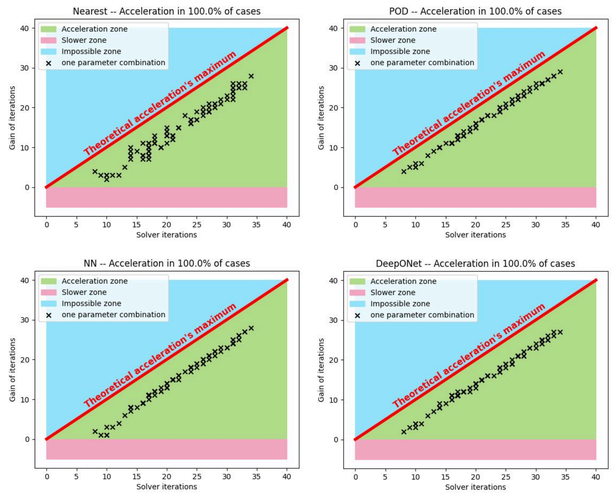
• Deep Operator Networks (DeepONet) [3]

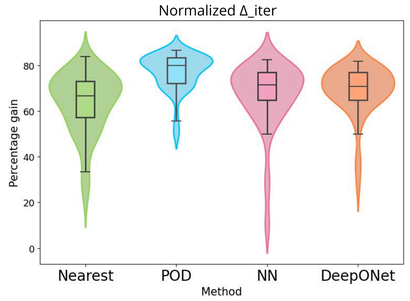
Neural operator architecture to approximate mappings between functions.
Supported by the **Universal Approximation Theorem for operators** [Chen & Chen, 1995].



Results

• 1D Nonlinear Poisson equation

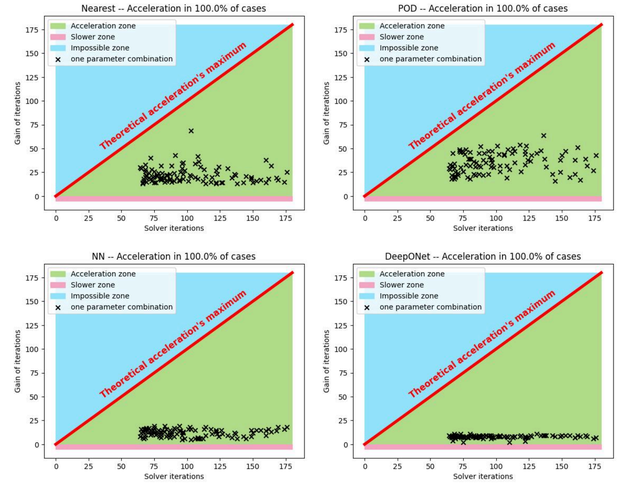


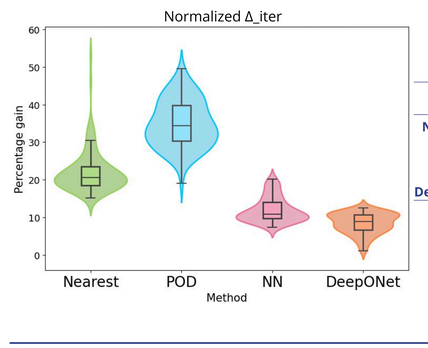


	Mean	Median
Nearest	63.47	66.67
POD	77.13	80.00
NN	66.58	71.43
DeepONet	68.89	70.83

n. obs = 100

• 2D calendering process





	Mean	Median
Nearest	22.16	20.54
POD	34.89	34.31
NN	12.06	10.91
DeepONet	8.49	8.89

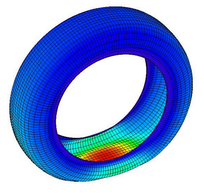
n. obs = 100

Preliminary conclusions

- **All initialization methods perform better** than using a constant field.
- **POD consistently outperforms** all other tested methods.
- In 1D, the iteration gain grows with baseline cost; in 2D the iteration gain is constant.
- Why not PINNs ? Longer training time, inefficient for parametric studies

Future work

- Extension to industrial test cases
- Generalization to geometrical variability



How to assess confidence in surrogate-based initialization ?

References

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